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CONDITIONAL AND UNCONDITIONAL HIGHER MOMENT CAPM: A COMPARATIVE STUDY

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ABSTRACT

The traditional Capital Asset Pricing Model, because of its conceptual and empirical disadvantage, needed some possible extension which led to the inclusion of higher moments in the model. Past studies showed that higher moments i.e. skewness and kurtosis, contributed to the risk premium of an asset. In the present study an attempt was made to compare unconditional and conditional higher moment Capital Asset Pricing Models and find the most suitable among these in context of Indian stock market. For illustrating the better model, the data of the companies listed in S&P BSE 500 Index has been considered. Akaike Information Criteria and Bayesian Information Criteria values were used for the selection of better model among these two models. The results revealed that conditional higher moment model gave better results as compared to unconditional higher moment model.

Keywords: Skewness, kurtosis, Akaike Information Criteria, Bayesian Information Criteria.

1. INTRODUCTION

William Sharpe (1994) proposed the Capital Asset Pricing Model (CAPM) that described the relationship between the expected return and risk related to an asset. CAPM assumed that a positive correlation exists between the return on an asset and the risk (beta coefficient) related with that return. The systematic risk or market risk (represented by â) was considered as an important factor by CAPM for the assessment of the asset price. CAPM assumed that a positive linear relationship exists between the asset's return and systematic risk which could be sufficient to explain the cross-sectional returns.

CAPM had various application, it not only assigns capital for machineries and factories (real investment) but could also be used to assigns funds for bonds and stocks (financial investment) etc. It could also be used for taking decisions related to the evaluation of a portfolio performance, investment capital expenditure, financing and corporate restructuring. The guidelines related to the fair price issue could be provided with the help of CAPM. It could also be used for the determination of the expected rate of return on a stock and to judge whether the stock is overvalued or undervalued. While studying about capital market efficiency, the relationship between risk and return is considered as an important key point.

One of the important assumptions considered by CAPM was that the assets with higher risk will yield higher returns than the assets with lower risk. The Sharpe (1964) and Lintner's (1965) version of CAPM assumed that the capital market is perfect i.e. at the same interest rate, an investor can borrow or lend money. The other assumptions for the CAPM are identical expectation of investors with respect to assets (homogeneity), absence of the transaction cost and taxes, normal distribution of returns and the price of a stock could not be influenced by individual alone.

The version of CAPM developed by Sharpe and Lintner considered that the expected rate of return is related to the systematic risk. One of the main assumption for CAPM to hold was that the returns should be normally distributed which is crucial assumption. Various studies showed the returns' distribution was non-normal especially in high frequency data. Also there were some drawbacks of the model which caused the researchers to focus their attention towards the higher moments i.e. skewness and kurtosis (the third and fourth moment).

Kraus and Litzenberger (1976) and others tested the extended form of CAPM i.e. the three moment CAPM by including the coskewness in asset valuation models and obtained mixed results. The conditional skewness captured asymmetry in risk, particularly downside risk and explained the cross-sectional variation of returns. The four moment CAPM was derived by Fang and Lai (1994) which incorporated cokurtosis besides the covariance and coskewness in the model. The fourth moment i.e. cokurtosis well explained the generating process of return in future markets.

The econometric equation of the **Unconditional Higher Moment CAPM** using the **Generalized Method of Moments (GMM)** specification is given as:

$$R_{it} = a + \hat{\beta}_{imt}(R_{mt}) + \hat{\gamma}_{imt}(R_{mt})^2 + \hat{\delta}_{imt}(R_{mt})^3 + \mu_t$$

where R_{it} denotes the excess return on stock i over month t,

a denotes the constant term,

R_{mt} denotes the excess market return over month t,

$\hat{\beta}_{imt}$ denotes the systematic risk (beta) i.e. covariance,

$\hat{\gamma}_{imt}$ and $\hat{\delta}_{imt}$ denotes the coskewness (lambda) and cokurtosis (gamma) respectively.

For the estimation using GMM, excess market return and lagged excess market returns were used as instrumental variables.

The introduction of the **Autoregressive Conditional Heteroscedasticity /Generalized Autoregressive Conditional Heteroscedasticity (ARCH/ GARCH)** process by Engle (1982) and Bollerslev (1986) gave importance to the testing, estimation and modeling of time varying volatility and conditional CAPM. The model tested using ARCH process provided better evidence of the risk-return relationship. The conditional model of Pettengill, Sunderam and Mathur (1995) examined the risk-return relationship and provided stronger results.

The econometric equation of the **Conditional Higher Moment CAPM** using the following **GARCH(1,1)** specification is given as:

$$E_{t-1}(R_{it}) = a + \hat{\beta}^{c}{}_{imt}\, E_{t-1}(R_{mt}) + \hat{\gamma}^{c} i_{mt}\, E_{t-1}(R_{mt})^2 + \hat{\delta}^{c}{}_{imt}\, E_{t-1}(R_{mt})^3 + \mu_t$$

Where R_{it} denotes the excess return on stock i over month t,

a denotes the constant term,

R_{mt} denotes the excess market return over month t,

$\hat{\beta}_{imt}$ denotes the systematic risk (beta) i.e. conditional covariance,

$\hat{\gamma}_{imt}$ and $\hat{\delta}_{imt}$ denotes the conditional coskewness (lambda) and conditional cokurtosis (gamma) respectively.

The skewness and kurtosis could not be diversified by increasing the size of the portfolio, hence, this non diversifiable property skewness and kurtosis became important to be considered for valuation of the asset. Earlier in CAPM it was assumed that the risk related with an asset doesn't differ with respect to time. Later it was obtained by the researchers that the covariance, coskewness and cokurtosis risk vary with respect to time and so are their prices, which suggested the relationship between coskewness and cokurtosis vary with respect to time.

2. LITERATURE REVIEW

Andor *et al.* (1999) performed the study using the CAPM for the Hungarian capital market using regression technique for analyzing the company's risk and their average return. The data of 17 Hungarian companies listed in

Budapest Stock Exchange for the period from July 1991 to June 1999 was used in the study and concluded that there existed positive correlation between beta and actual returns. In other words, CAPM was obtained suitable for the Hungarian capital market by them.

Gunnlaugsson (2004) performed the study to check the applicability of CAPM for Icelandic stock market using **Ordinary Least Square (OLS)** for estimating the alpha and beta coefficients as well as nonsystematic risk. Using the monthly data of 27 stocks listed in Iceland Stock Exchange (ICEX) for the period from January 1990 to May 2004, it was proved that a strong relationship exists between the risks and the stock returns. Also it was concluded by them that higher (lower) risk yields higher (lower) returns and CAPM works effectively for the ICEX.

Michaildis *et al.* (2006) performed the study to test the applicability of CAPM on Greek security market. Using the weekly data of 100 companies listed on the Athens Stock Exchange (ASE) for the period from January 1999 to December 2002, it was concluded by them that CAPM was not applicable for ASE as the theory of higher (lower) risk yielding higher (lower) returns was not achieved for the same.

Refai (2009) performed the study to establish the relationship between risk and returns in the Jordan Stock Exchange. OLS regression was used to estimate the risk and the version of M-GARCH for time-varying risk. Using the monthly data for the period from December 1999 to October 2008, it was obtained that CAPM might not be applicable for the Jordan Stock Exchange as time-varying risk obtained by M-GARCH shows greater variance as compared to OLS beta and the hypothesis of (positive) relationship between risk and return was being rejected.

Paul and Asarebea (2013) performed the study to test the applicability of CAPM in India with reference to National Stock Exchange (NSE) using the monthly data of 5 companies listed in NSE for the period from 2005 to 2009. It was obtained by that CAPM adequately explained the risk return relationship and it validates the theory that high (low) risk yields high (low) returns.

Shamim *et al.* (2014) performed the study to test the applicability of the CAPM for Karachi Stock Exchange (KSE). The **Augmented Dickey Fuller (ADF)** unit root test was used for testing and making the series stationary. Using the data of companies from different sectors listed in KSE for the period from 2008 to 2012 and applying the OLS regression for the estimation of risk coefficients and paired t-test for the comparison between actual and expected returns, it was obtained that CAPM is not valid for

KSE as single risk factor model is not sufficient to predict the expected return accurately.

Choudhary and Apoorva (2018) performed the study to test the applicability of CAPM in Indian stock market. Using OLS regression for estimation of risk coefficients for the monthly data of five PSU's listed in **Bombay Stock Exchange (BSE)** for the period from January 2016 to December 2017, it was concluded that the CAPM is not applicable for BSE and also the theory that high (low) risks yields high (low) returns was contradicted by the study.

Xiao *et al.* (2019) performed the study to test the effectiveness of CAPM in China with reference to Shanghai Stock Exchange using the two tests: time series test and cross sectional test. Using the weekly data of 18 industries for the period from June 2016 to December 2018 it was concluded that CAPM is not effective for Shanghai Stock Exchange.

The objective of the present study is to estimate and compare the unconditional and conditional higher moment CAPM models for the Indian stock market and to find the better model between the two.

3. DATA DESCRIPTION AND METHODOLOGY

The data considered for the present study is secondary in nature which consists of the monthly returns data of 11 stocks listed in S&P BSE 500 Index for the period from January 1993 to March 2015. Only those stocks have been considered whose data was available for the entire period. The discrete monthly returns of the stocks were taken as the dependent variable and the market return as the independent variable.

For the present study, the higher-moments, beta, lambda and gamma i.e. covariance, coskewness, and cokurtosis risks were estimated. Before estimating the model, firstly, the normality test i.e. **Kolmogorov Smirnov (KS)** test was done to examine the normality of the residuals for which the results showed that the distribution of residuals is not normal. Hence the use of GMM is appropriate. Secondly, to check whether the series is stationary or not, the unit root test using **ADF** test was done. For detecting the presence of ARCH effects the Heteroscedasticity test was carried out. The process of model validation was carried out which showed that the models fitted achieved its intended purpose. Finally the model selection was done using the two important information criteria namely **Akaike Information Criteria and Bayesian Information Criteria (AIC and BIC).**

In the first model i.e. **Conditional Higher Moment CAPM**, for the estimation of conditional higher-moments, GARCH (1,1) specification was used to obtain the result. In the second model i.e. **Unconditional Higher Moment CAPM**, on a sample of 11 stocks, simple regression was applied using GMM as the estimation technique. Then on the results obtained from regression, rolling regression using 60 months rolling window with step size 1 was used to obtain the rolling coefficients.

Rolling regression is a process often used in time series to test the stability of the model parameters with respect to time by applying different sampling periods. While running the rolling regression, a **window size** and **step size** has to be chosen. The number of consecutive data points to be used in each sample is known as window size while the number of periods a window has to be advanced is known as step size.

GMM is a method used for estimating the parameters considered in model. When the probability distribution is known, **Maximum likelihood (ML)** estimation gave feasible results. However, in many cases, this dependency becomes the weakness when the distribution is unknown and the results obtained from ML are infeasible. GMM, which is also known an asymptotic distribution free approach for the estimation of the parameters, is preferred over ML estimation as GMM provides a way to estimate the parameters merely based on the information concluded from the given model.

Bollerslev (1986) and **Taylor (1986)** had independently introduced GARCH process which allowed the conditional variance to depend on its past lags. GARCH model is an extension of ARCH model. The GARCH model is considered to be more parsimonious than ARCH model as it gives better prediction and also permits wide range of behavior i.e. volatility.

Model validation is a process which compared the model output to the independent real world observations. In other words, a process known as model validation was used to confirm whether the model fulfils its predetermined purpose or not. Model validation was done by comparing the output attained by the model to another dataset termed as independent experimental dataset. This experimental dataset is also known as **train dataset** and the other one is known as **test dataset.**

Train dataset is the sample (independent) of data used for fitting the given model while the test dataset is a subset used to test the appropriateness of the model fitted for the trained dataset. The data used in the train dataset cannot be considered in the test dataset. The percentage usually taken for the test and train data set is 66:33, 70:30 or 80:20. In some studies 50:50 is also considered.

AIC and BIC are the techniques used for the evaluation of goodness of fit for the model as well as selecting the best model among the fitted different models. AIC is used to judge the quality of the model under consideration, related to every other model fitted for same set of observations while BIC is used for model selection based on information theory but within the Bayesian context. The difference between BIC and AIC is that BIC imposed greater penalty for the number of parameters than AIC. The smaller value of AIC and BIC indicates the better model among the models under consideration.

4. RESULTS AND ANALYSIS

For testing the normality, the linear model using OLS was fitted and then the normality of residuals was administered using KS test. The p-value obtained by using KS test showed that the residuals are non-normally distributed (**table 1**). Since the residuals are non- normally distributed, hence GMM is preferable over OLS method for the estimation of the parameters.

Table 1: Table representing different test statistics for checking the assumptions for fitting of the Unconditional and Conditional Higher Moment CAPM

Variables	*Kolmogorov-Smirnov Test for Normality*		*ADF Test for Stationarity*		*Heteroscedasticity test for ARCH effect*	
	Statistic	*p-value*	*t-Statistic*	*p-Value*	*R-square*	*p-Value*
Stock 1	0.139	<0.001	-16.531	<0.001	0.066	0.097
Stock 2	0.106	<0.001	-15.474	<0.01	2.416	0.020
Stock 3	0.092	<0.001	-17.053	<0.001	0.01	0.019
Stock 4	0.086	<0.001	-15.716	<0.001	0.019	0.090
Stock 5	0.09	<0.001	-14.996	<0.001	3.624	0.057
Stock 6	0.114	<0.001	-14.639	<0.001	1.97	0.061
Stock 7	0.178	<0.001	-16.259	<0.001	15.036	<0.001
Stock 8	0.078	<0.001	-19.189	<0.001	4.882	0.027
Stock 9	0.094	<0.001	-16.582	<0.001	0.663	0.016
Stock 10	0.051	0.041	-19.047	<0.001	7.049	0.008
Stock 11	0.041	0.043	-13.68	<0.001	7.686	0.006

For verifying the stationarity of all the 11 stocks, ADF test was used. The p-value for the ADF test showed the absence of unit root for (**table 1**). Hence, all the 11 stocks are stationary at level.

For detecting the presence of ARCH effects for all the 11 stocks, the heteroscedasticity test for ARCH was used. The p-value obtained by using the Heteroscedasticity test for all the stocks shows the presence of ARCH effects (**table 1**). Hence the use of GARCH(1,1) model is appropriate. Here only one lag is considered as the inclusion of higher lags did not yield parsimonious results.

To estimate the conditional higher moments i.e. covariance, coskewness and cokurtosis risks (beta, lambda and gamma) for each stock, the regression was fitted using GARCH(1,1) as the estimation technique. The entire constant and slope coefficients are displayed below (**table 2)**.

Estimates of the unconditional higher moments i.e. covariance, coskewness and cokurtosis risks (beta, lambda and gamma) for each stock, the regression were fitted using GMM as the estimation technique. Rolling regression using 60 months rolling window was used to estimate the coefficients of beta, lambda and gamma. The entire constant and slope coefficients are displayed below (**table 2)**.

Table 2: Coefficients for Unconditional and Conditional Higher Moment CAPM

Variable	*Unconditional Higher Moment CAPM*				*Conditional Higher Moment CAPM*			
	α	β	γ	δ	α	β	γ	δ
Stock 1	0.036	124.001	-135.383	-9407.329	0.031	0.758	-3.732	7.377
Stock 2	0.018	-105.085	106.485	8082.756	-0.001	0.970	-3.437	-15.981
Stock 3	-0.027	30.247	-28.335	-2245.864	-0.002	0.744	0.260	12.110
Stock 4	-0.049	-54.036	65.459	4206.578	-0.007	0.946	-0.061	-7.722
Stock 5	0.016	-62.789	65.539	4886.743	-0.015	1.026	-0.548	3.399
Stock 6	0.024	-158.982	153.928	12224.684	0.015	0.798	-2.791	20.239
Stock 7	-0.018	34.503	-37.302	-2578.604	-0.010	0.459	0.655	5.439
Stock 8	0.020	-109.547	98.632	8479.963	-0.017	1.333	-1.204	-12.623
Stock 9	-0.076	81.495	-63.307	-6150.365	-0.010	0.393	2.906	26.963
Stock 10	0.015	90.721	-48.999	-6829.787	-0.030	1.062	1.093	-0.453
Stock 11	-0.014	-2.592	2.597	277.437	-0.005	0.980	-1.411	-7.107

For the **Unconditional Higher Moment CAPM,** the intercept term or alpha is the return on investment that is not result of general movement in market. An alpha of zero in CAPM indicates that the particular portfolio is being tracked by benchmark index and no additional value has been added or lost in comparison to the broad market. The alpha values of the stocks were found to be either insignificant or different from zero. The slope term or beta in CAPM is a measure of the volatility or systematic risk of any portfolio in comparison to the market. The negative beta (beta < 0) indicates inverse relation with the market. The beta values of 6 stocks (Stocks 2, 4, 5, 6, 8 and 11) were found to be negative and insignificant. The coskewness in CAPM is used to measure the risk related to an asset with regards to market risk whereas the cokurtosis in CAPM measures the extreme positive and negative deviations at the same time. The results reported in the above table reveals that 5 stocks (Stock 1, 3, 7, 9 and 10) stocks showed negative value of skewness whereas 6 stocks (Stock 2, 4, 5, 6, 8 and 11) showed positive value of skewness. The positive skewness means the higher probability of assets having positive returns while negative skewness value means the higher probability of assets having lower returns simultaneously. The values of kurtosis for the 6 Stocks (Stock 2, 4, 5, 6, 8, 9 and 11) in the above table clearly showed that the stocks depict leptokurtic (kurtosis value > 3) behaviour also described as fat tails which means the data deviates from normality. The positive skewness/kurtosis in the model reduces the risk of considered stocks and also lowers the expected return. Similarly, for the **Conditional Higher Moment CAPM,** the alpha values of the stocks were found to be either insignificant or different from zero. The beta values of all the stocks were found to be significant. The results reported in the above table reveals that 5 stocks (Stock 4, 5, 6, 8 and 11) stocks showed negative value of skewness whereas 6 stocks (Stock 1, 2, 3, 7, 9 and 10) showed positive value of skewness. The values of kurtosis for the 5 Stocks (Stock 1, 3, 5, 6, 7 and 9) showed leptokurtic behaviour meaning that the data deviates from normality.

After estimation of the constant and slope coefficients for all the 11 stocks by the two models described, the process of model validation was carried out using test and train datasets. The train dataset contained 70% of the data while the test data contained 30% of the dataset. The model perceived and learned from the trained dataset which is used to train the model. The test dataset is used only when the model is trained and is used to obtain an unbiased assessment of the final model. When the model fitted on the trained dataset also fits the test data well, we can say that minimal overfitting occurred. The graphical representation of both the models (**Appendix 1 and Appendix 2**) showed that the result of Conditional Higher Moment CAPM is better in comparison to the Unconditional Higher Moment CAPM. The **Mean Standard Error (MSE)**, is a measure of the amount of error in given statistical model, measures the average squared difference between the observed and predicted values. MSE equals zero when a model has no error but as the model error increases, MSE increases. MSE calculated for both the models are listed in the table below (**table 3).**

Table 3: MSE for Unconditional and Conditional Higher Moment CAPM

	Train dataset		*Test dataset*	
Variables	*Unconditional Higher Moment CAPM*	*Conditional Higher Moment CAPM*	*Unconditional Higher Moment CAPM*	*Conditional Higher Moment CAPM*
Stock 1	26.063	0.04	16.216	0.015
Stock 2	19.386	0.021	10.253	0.006
Stock 3	1.387	0.022	8.292	0.008
Stock 4	6.695	0.013	48.371	0.012
Stock 5	6.606	0.024	43.372	0.014
Stock 6	47.902	0.038	31.436	0.01
Stock 7	1.936	0.012	13.844	0.005
Stock 8	21.342	0.037	12.249	0.009
Stock 9	10.667	0.025	69.843	0.04
Stock 10	14.473	0.095	82.278	0.037
Stock 11	0.044	0.004	0.249	0.002

The result in the above tables clearly showed that the MSE values obtained for both the datasets are lower for the Conditional Higher Moment CAPM than the Unconditional Higher Moment CAPM. The lower the value of MSE, the better is the model.

Finally the model selection was then carried out by means of AIC and BIC values. The AIC and BIC values for all the 11 stocks are listed below (**table 4)**. The criterion to choose the best model between the two is to choose the one with minimum AIC/BIC value.

Table 4: Coefficients of AIC and BIC for model selection

Variables	*Statistics*	*Train dataset*		*Test dataset*	
		Unconditional Higher Moment CAPM	*Conditional Higher Moment CAPM*	*Unconditional Higher Moment CAPM*	*Conditional Higher Moment CAPM*
Stock 1	AIC	11.312	2.245	-0.380	-1.569
	BIC	12.828	4.270	-0.259	-1.360
Stock 2	AIC	12.542	-1.302	-1.049	-2.464
	BIC	14.058	2.722	-0.928	-2.256
Stock 3	AIC	13.096	4.590	-0.952	-1.931
	BIC	14.612	6.614	-0.831	-1.723
Stock 4	AIC	9.904	4.949	-1.479	-1.509
	BIC	11.420	6.974	-1.358	-1.300
Stock 5	AIC	12.535	4.428	-1.058	-1.461

	BIC	14.051	6.453	-0.937	-1.252
Stock 6	AIC	12.100	6.285	-0.678	-1.861
	BIC	13.616	8.310	-0.557	-1.653
Stock 7	AIC	12.651	0.518	-1.631	-2.392
	BIC	14.167	2.542	-1.510	-2.183
Stock 8	AIC	12.047	2.706	-0.391	-1.902
	BIC	13.563	4.730	-0.270	-1.693
Stock 9	AIC	9.349	5.407	-0.128	-1.554
	BIC	10.865	7.432	-0.007	-1.346
Stock 10	AIC	11.083	5.792	0.386	-0.700
	BIC	12.599	7.817	0.507	-0.492
Stock 11	AIC	8.146	2.874	-2.606	-3.420
	BIC	9.662	4.898	-2.485	-3.212

The results in the above table showed that AIC/BIC values obtained for both the datasets was minimum for **Conditional Higher Moment CAPM** as compared to **Unconditional Higher Moment CAPM.** The lower the AIC/BIC value, the better is the model.

5. CONCLUSION

There were some alterations done in the traditional CAPM that measured the relationship between the risk and return. The altered model consisted of the higher moments i.e. covariance, coskewness and cokurtosis (beta, lambda and gamma). The present study focused mainly on the comparison and selection of the best model between the two models (Unconditional Higher Moment CAPM and Conditional Higher Moment CAPM). Monthly data of the stocks listed in S&P BSE 500 index for the period from January 1993 to March 2015 was used for the study.

Consideration of the higher moments in the model was done to study the detailed analysis of single risk factor instead of identifying more risk factors as the investors were concerned more about the higher moments of the returns distribution. Out of the two higher moment models estimated, the model selection was carried out by means of AIC/BIC. The minimum AIC/BIC values were obtained for the Conditional Higher Moment CAPM as compared to Unconditional Higher Moment CAPM. Hence, the **Conditional Higher Moment CAPM** is considered as the best model.

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www.rbi.org.in/Scripts/BS_NSDPDisplay.aspx?param=4) websites which is publically accessible.

Declarations: The authors declare no conflict of interests.

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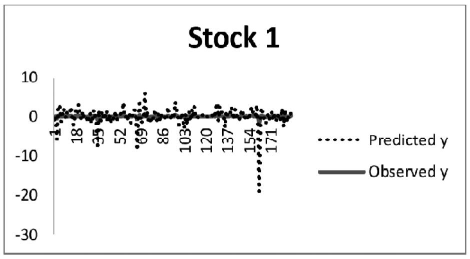
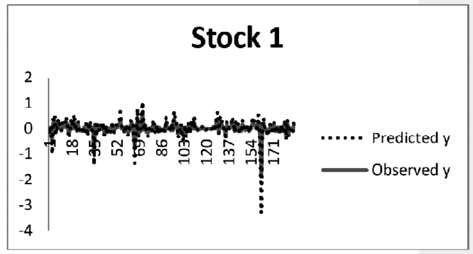
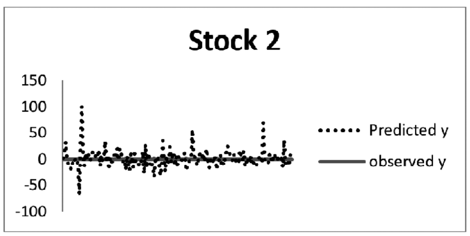
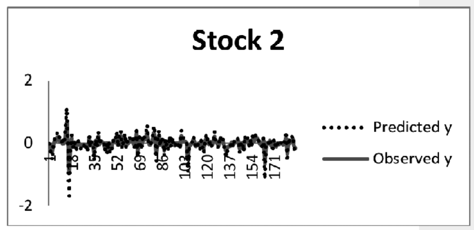
Appendix 1
Graphical representation of Train dataset

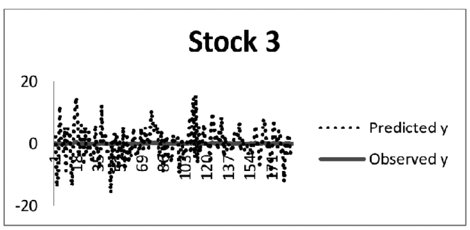
Unconditional HM CAPM | Conditional HM CAPM

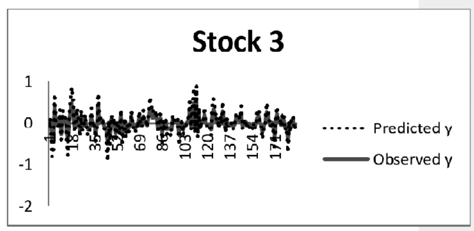


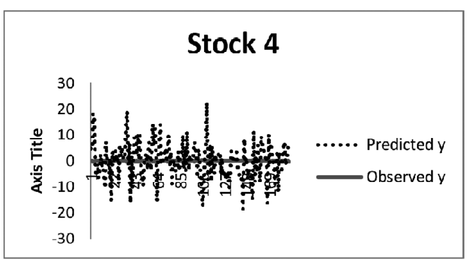


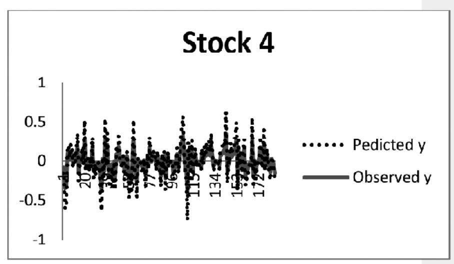


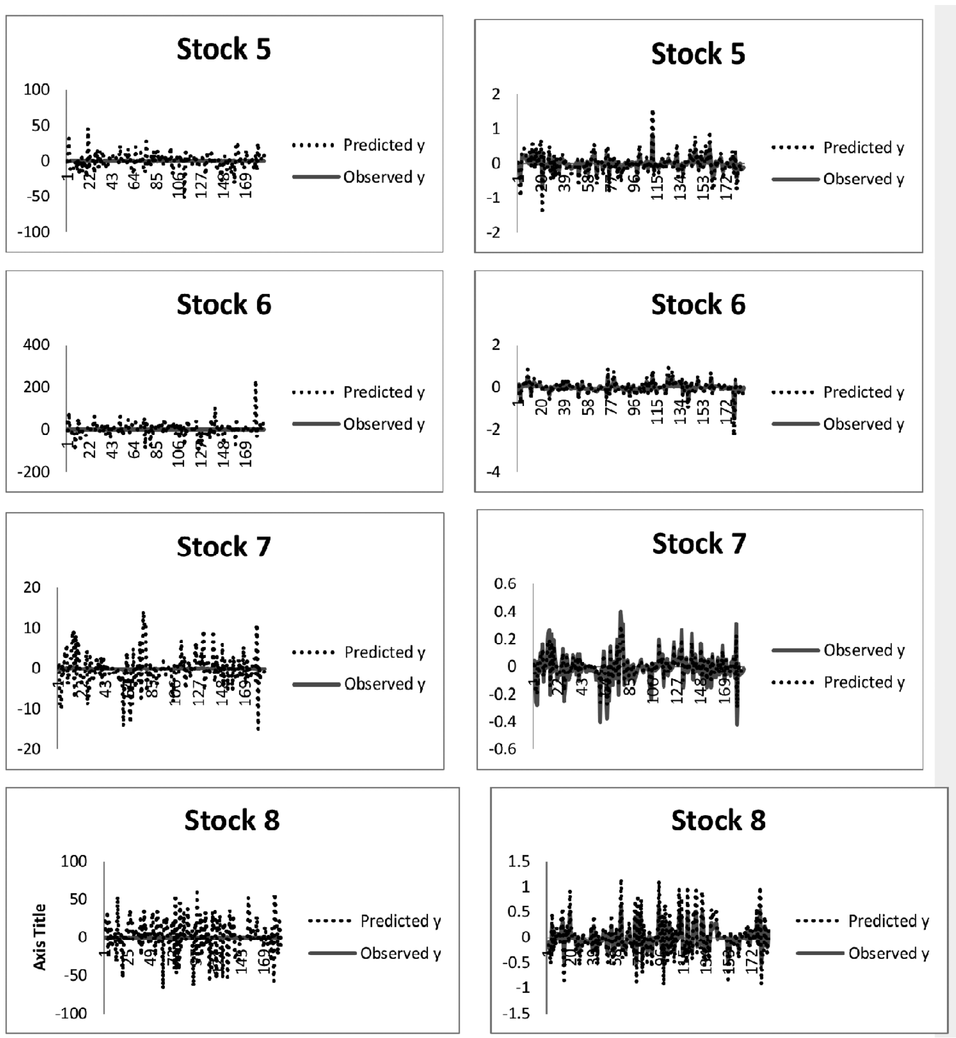


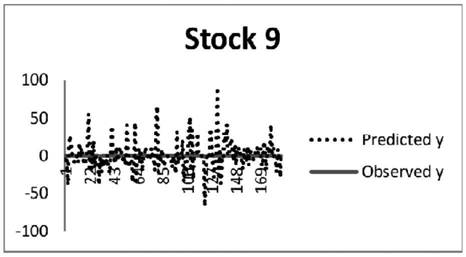


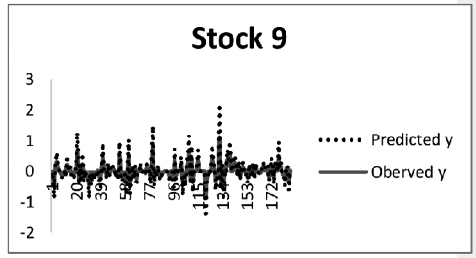


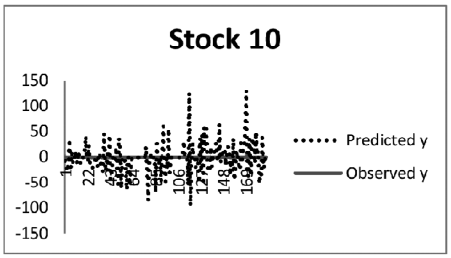





Stock 5
Predicted y
Observed y
Stock 5
Predicted y
Observed y
Stock 6
Predicted y
Observed y
Stock 6
Predicted y
Observed y
Stock 7
Predicted y
Observed y
Stock 7
Observed y
Predicted y
Stock 8
Axis Title
Predicted y
Observed y
Stock 8
Predicted y
Observed y


Stock 9
100
50
0
-50
-100
Predicted y
Observed y


Stock 9
3
2
1
0
-1
-2
Predicted y
Observed y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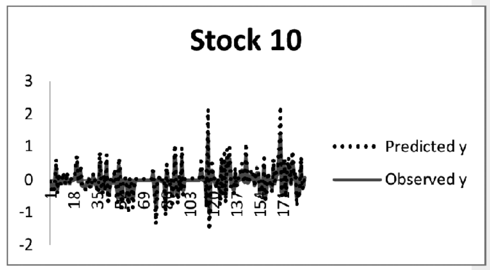
100
50
0
-50
-100
-150
Predicted y
Observed y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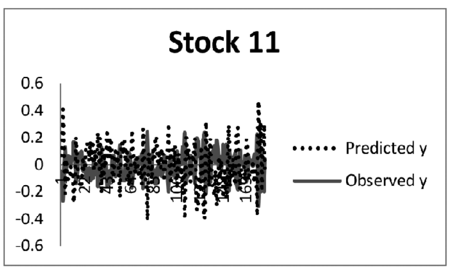
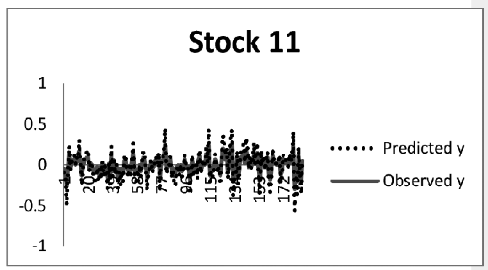
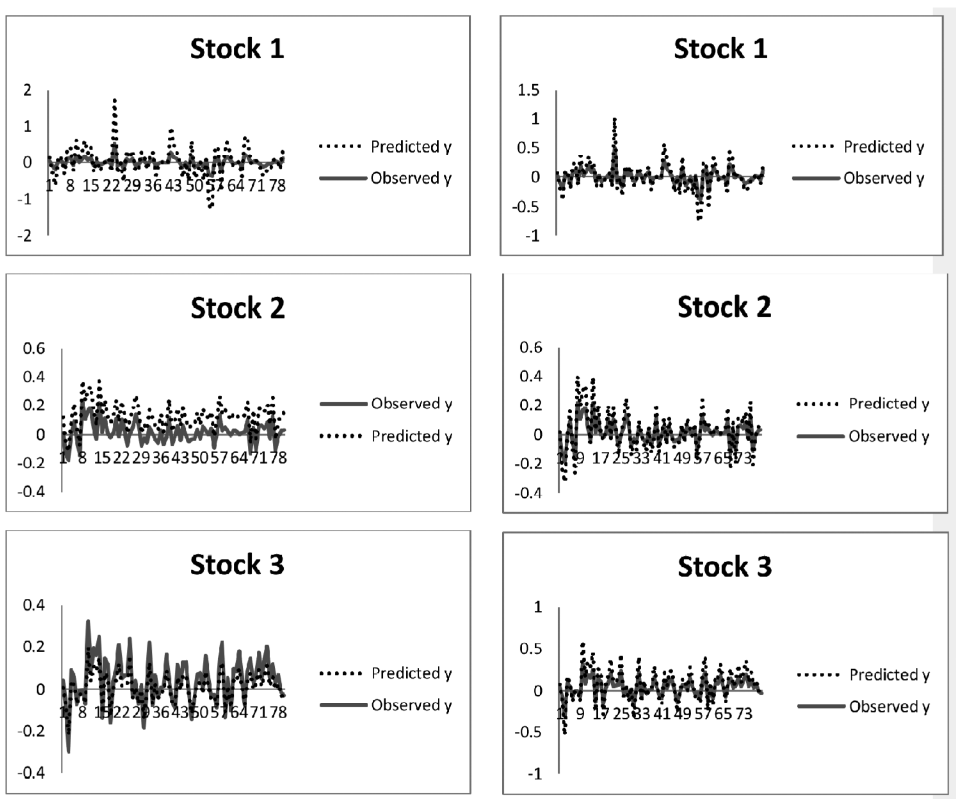
3
2
1
0
-1
-2
Predicted y
Observed y

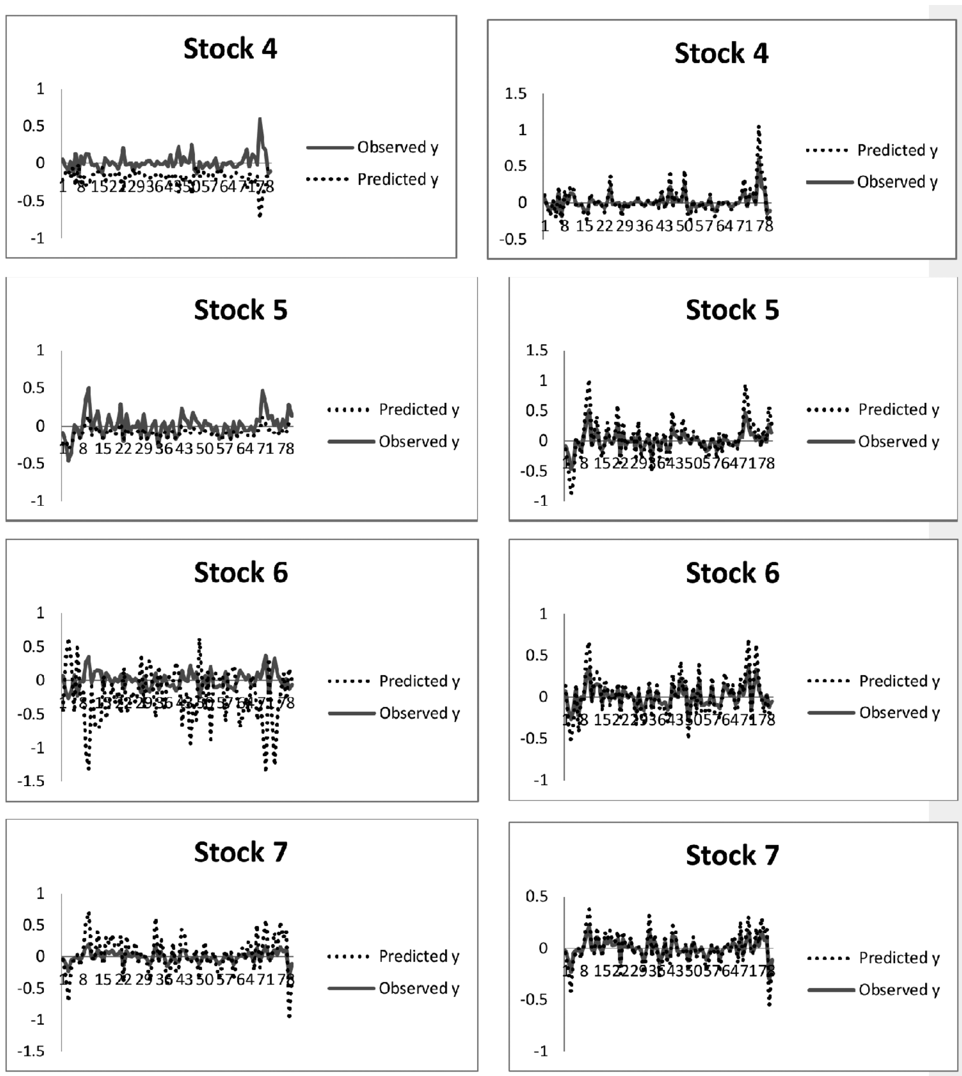

Stock 11
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
Predicted y
Observed y

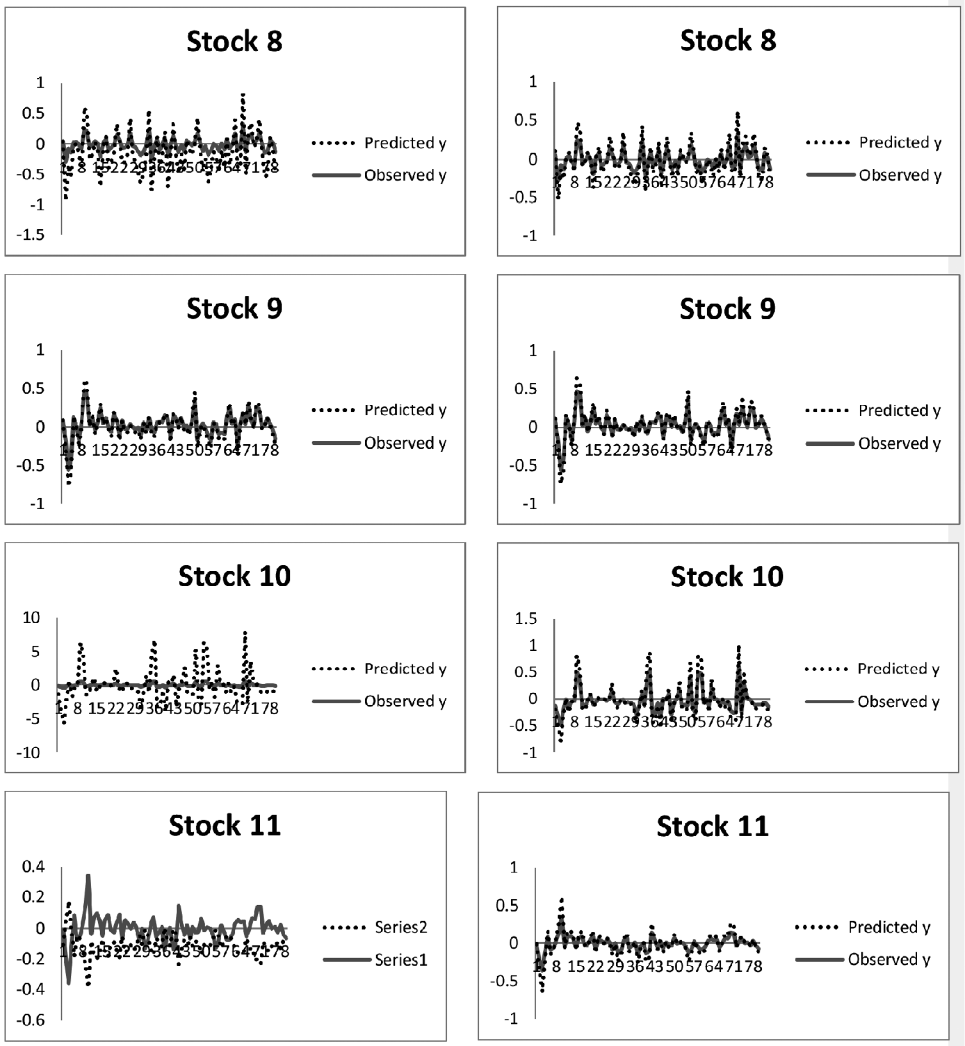

Stock 11
1
0.5
0
-0.5
-1
Predicted y
Observed y

Appendix 2
Graphical representation of Train dataset

Unconditional HM CAPM | Conditional HM CAPM




Stock 4
Observed y
Predicted y
Stock 4
Predicted y
Observed y
Stock 5
Predicted y
Observed y
Stock 5
Predicted y
Observed y
Stock 6
Predicted y
Observed y
Stock 6
Predicted y
Observed y
Stock 7
Predicted y
Observed y
Stock 7
Predicted y
Observed y


Stock 8
Predicted y
Observed y
Stock 8
Predicted y
Observed y
Stock 9
Predicted y
Observed y
Stock 9
Predicted y
Observed y
Stock 10
Predicted y
Observed y
Stock 10
Predicted y
Observed y
Stock 11
Series2
Series1
Stock 11
Predicted y
Observed y